Small oscillations problem

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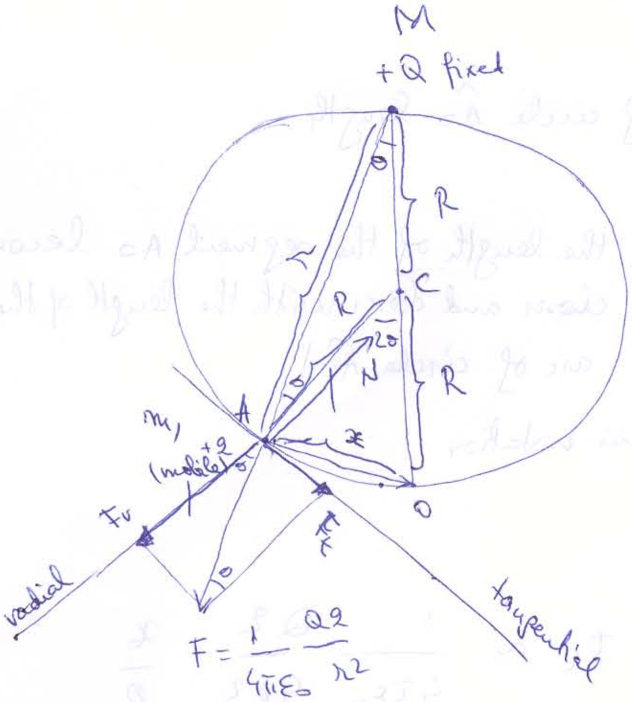

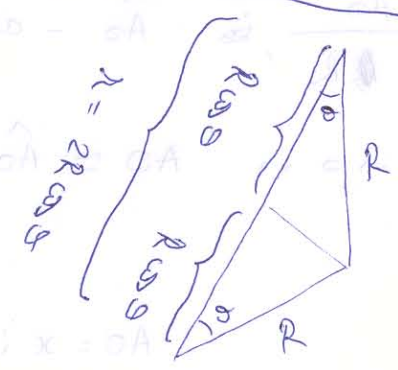

+q forced to move on a circle ⇒ any force will be decomposed in a radial component that will be canceled by the normal from the circle (imagine +q moves in a tunnel...) and a tangential component that will create oscillatory motion.

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$$

$F_t = F \cdot \sin\theta$ (toward equilibrium point)

as $\theta \to 0 \Rightarrow A \to O \Rightarrow (\widehat{F_t, \hat{AO}}) \to 0$.

For small oscillations (harmonic oscillations) ⇒ $\underline{\theta \simeq 0 \text{ is the approximation needed}}$

If $AO = x$, an oscillatory motion would be given by a force: $F = -k \cdot x$

So, as $\theta \to 0$, our F_t will become close to $F = -kx$. We will identify k and then T, the period of small oscillations, will be:

$$T = 2\pi\sqrt{\frac{m}{k}}, \quad m = \text{mass of } +q$$

so what is k?

So we need to find exactly $F_t = f(R, m, Q, q, \theta)$ and then do $\theta \to 0$.

$$F_t = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qq}{4R^2\cos^2\theta}\sin\theta \quad \Bigg\| \Rightarrow \boxed{F_t \sim \frac{1}{4\pi\varepsilon_0}\frac{Qq}{4R^2}\cdot\theta} \quad (1)$$

$\theta \to 0 \Rightarrow \sin\theta \sim \theta$

$\cos\theta \sim 1$

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by definition of radian =>

$$\theta = \frac{\hat{AO}}{R} \quad \text{in} \quad \hat{AO} - \text{arc of circle } \hat{AO} \text{ length}$$

and as $\theta \to 0$ => $AO \simeq \hat{AO}$ (the length of the segment AO becomes closer and closer with the length of the arc of circle $\hat{AO}$).

$AO = x$ in our notation

$$\boxed{\theta \simeq \frac{x}{R}}$$

$$F_t \simeq \frac{1}{4\pi\varepsilon_0}\frac{Q q}{4R^2}\cdot\theta$$

$$\Rightarrow F_t \simeq \frac{1}{4\pi\varepsilon_0}\frac{Qq}{4R^2}\cdot\frac{x}{R}$$

We add a "$-$" sign (toward equilibrium point)

$$\boxed{F_t \simeq -\underbrace{\left(\frac{1}{4\pi\varepsilon_0}\frac{Qq}{4R^3}\right)}_{k} x}$$

We see it is indeed of the form $F_t \sim -kx$ (harmonic motion)

=> we identify $\boxed{K = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{4R^3}}$

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \left(k = m\omega^2 \Rightarrow \omega = \sqrt{\frac{k}{m}},\ T = \frac{2\pi}{\omega}\right)$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{\frac{1}{4\pi\varepsilon_0}\frac{Qq}{4\pi R^3}}}}$$

$$\boxed{T = 2\pi\sqrt{\frac{m\,4\pi\varepsilon_0\cdot 4\pi R^3}{Qq}}}$$

Montreal, 18 mai 2008,
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